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On the relation between linear stability analysis and mean flow properties in wakes.

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In recent studies on wake stability [1–3], it has been observed that a simple linear stability analysis applied to the mean flow instead of the basic flow, could give an accurate prediction of the global mode selected frequency, although these phenomena are strongly non-linear. In this letter, we study the transient regime between the stationary (so called basic state) and unstationary solutions of the wake of a circular cylinder at Re=150. We show that the shift of the global frequency as a function of time due to strong non-linear effects, can be interpreted by a continuous mean flow correction induced by the growth of the instability. We show that during this transient regime, the mean state as a function of time plays the role of an instantaneous basic state on which the global frequency can be determined linearly.

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It is known that most unstable flows such as shear layers, jets and wakes present a strong non-linear dynamic even near the threshold of instability. For synchronized open flows, like bluff body wakes, these behaviors can be described by a Landau or Ginzburg-Landau model [4], directly linking the non-linear frequency and amplitude of the perturbation with the distance between the control parameter and its critical value at the threshold [5].

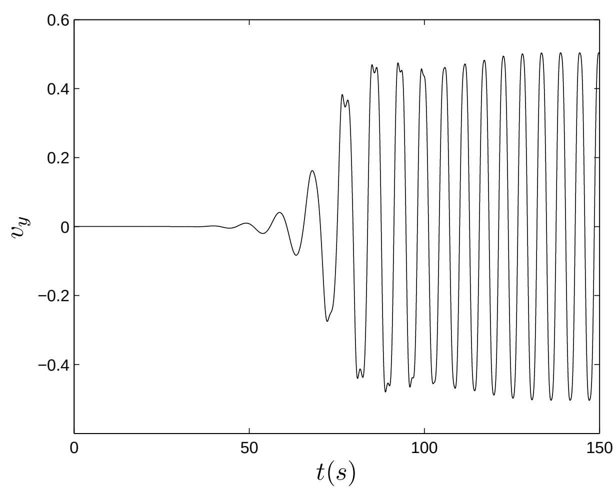

FIG. 1: Time evolution of the transversal velocity component measured at $y = 0$ during the transition from the steady to saturated state.

However, recent results have shown that the best theoretical predictions describing the behavior of such flows were based on a fully linear theory but applied to a mean state (i.e. time averaged unstationnary flow) instead of the usual basic steady state [3]. Equivalent results using this technique had already been obtain by [6] on directly measured mean velocity profiles but no comment was made of the fact that the mean flow gave good frequency predictions. This important result have also been confirmed recently in [1] using this time a global linear stability analysis. With this analysis, it was shown that the solution obtained from the mean flow was marginally stable (i.e the growth rates of the eigenvalue computed were found to be almost zero). Similar conclusions have been obtained for forced open flows (wake behind an rotating oscillating cylinder) [2]. In this case, a significant change in the mean flow induced by the forcing conditions, have been observed. The consequences of this mean flow distortion were that the stability properties of the forced flow, and so the selected spatial modes, changed, following scaling laws as a function of the forcing parameters as can be observed for natural wakes [7] with the Reynolds number. In recent studies, Thiria & Wesfreid [2] have conjectured that the strong non-linearities generated by these unstable flows are concentrated in this mean flow distortion. The mean state would play the role of a new basic state where linear modes (defined by their frequency, amplitude and spatial shape) are selected as a function of its properties. More precisely, it would mean that any changes in the mean configuration of the flow would be equivalent to a change in the se-

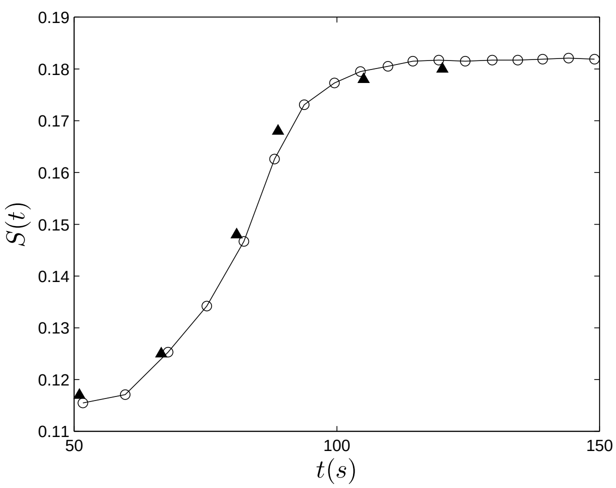

FIG. 2: Instantaneous Strouhal number as a function of time during the transient regime and comparison between numerical and theoretical results (which will be discussed later). Values denoted by ▲ correspond to global frequencies obtained by applying the linear saddle point criterion [10] on the instantaneous mean flow.

lected mode, which is selected linearly and fixed by the mean state. In the present letter, we study the transient regime of a flow past a circular cylinder at $Re = 150$ from its steady to saturated state: the flow is first forced to be symmetrical about the axis $y = 0$, a perturbation is then applied and the flow evolves and converges in time to its unstationary solution. We show that the non-linear shift of the frequency during this transient regime [5, 8], can be explained by a continuous distortion of the mean flow as a function of time, generated by the growth of the perturbation, and that this frequency can be predicted instantaneously by a linear stability analysis.
The flow past the circular cylinder at $Re = 150$ and then the dynamic of the transient regime have been obtained by numerical computation using the spectral finite element code $NEKTON$ [9] simulating an unconfined $2D$ wake. The cylinder has a diameter of $d = 1$ and the upstream velocity is $U_0 = 1$, so the viscosity ν is set to get a Reynolds number of $Re = 150$. The figure 1 displays a typical time evolution of the transversal fluctuating velocity component v_y during this transition showing that the selected period decreases to lower value as time increases (i.e. shift to higher frequency as a function of time). The instantaneous Strouhal number $St(t) = f(t)d/U_0$ has been extracted using a wavelet transform applied to the velocity signal $v_y(t)$ displayed in figure 1. The evolution of $S(t)$ is displayed in figure 2.

As can be seen, the Strouhal number evolves from $St = 0.116$ (the first frequency selected by the wake) to 0.182. This typical evolution illustrates how the global selected frequency is shifted due to the strong non-linear effects as the growth of the instability increases.
We also studied the evolution of the time dependant constructed mean flow by an ensemble average as a function of time during this transient regime . The information concerning this local mean state were obtained by computing 100 transient regimes for the entire flow field $(u(x, y), v(x, y))$ triggered each time by a controlled initial perturbation with a different phase. These different realizations then allowed access, by an ensemble average, to the instantaneous mean flow as a function of time [13]. This is displayed in figure 3. The evolution of the flow during the transient regime from the initial symmetric basic steady state to saturated state shows a drastic change in its mean properties due to strong non-linear effects as already observed [11] and shows clearly that the mean spatial shear layers are very different between the beginning and the end of the transient regime where the size of the recirculation bubble is reduced. This first observation supposes strongly that there are no obvious reasons to affirm that the instability which is observed for the developed wake, can be explained by simply studying the unstable modes on the basic state.

Then, the local stability properties of the mean flow, for each time, were obtained by solving numerically the inviscid Orr-Sommerfeld equation for the streamfunction $\Psi(x, y, t) = \int_0^y U(\eta)d\eta + \psi(x, y, t)$, where $\psi(x, y, t) = \mathcal{R}e\{\phi(y)e^{i(kx-wt)}\}$:

$$(kU(y) - \omega)(\phi'' - k^2\phi) - kU''(y)\phi = 0 \qquad (1)$$

with boundaries conditions $\phi(-\infty) = \phi(+\infty) = 0$, and where $U(y)$ is the local velocity profile depending on the transversal coordinate y, k and ω are respectively the complex wave number and the complex frequency of the perturbation and ψ the associated eigenfunction. It has been shown, [2, 6] that the use of an inviscid equation instead of the complete Orr-Sommerfeld equation was sufficient to determine the local properties of the flow even for low Reynolds numbers. Since we want to determinate the theoretical predicted global frequency, one has to differentiate a convective from an absolute local instability and then look for solutions verifying $\mathcal{D}(\omega, k) = 0$ and $\left(\frac{\partial \omega}{\partial k}\right)_{k_0} = 0$ [12] where $k_0 = k_r + ik_i$ and $\omega_0 = \omega_r + i\omega_i$ are respectively the complex absolute wave number and frequency. One has to note that, even if the global linear stability analysis as used in [1] is the more accurate, the choice of using local stability analysis in this paper is deliberate. It has been used in order to physically understand precisely the evolution of the stability properties as a function of time which is not possible with the technique cited above.
Local properties of the wake (absolute frequency $\omega_{0r}(x)$ and absolute growth rate $\omega_{0i}(x)$) during the transient regime are displayed in the figure 4 for the five instances in time chosen in figure 3. As can be seen, the absolute

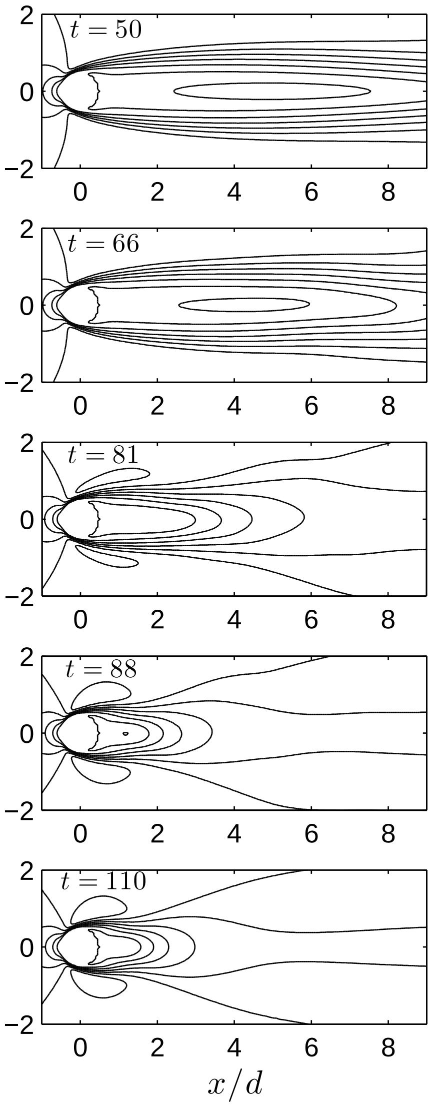

FIG. 3: Evolution of the mean flow as a function of time during the transitory represented by lines of isovalues of the velocity modulus. The first case corresponds to the basic flow while the last case corresponds to the time averaged flow of the fully developed street at $Re = 150$.

region ($\omega_{0i} > 0$), initially extends over 10 cylinder diameters when the wake is in its basic state decreases quickly with time and reaches its final value at saturation. This strong evolution can also be seen on the spatial distribution of absolute local frequencies $\omega_{0r}(x)$. As time increases, variations of ω_{0r} are faster and their local values globally increase in the absolute region.
The global frequency was then determinate by applying the saddle point criterion on the numerical results based

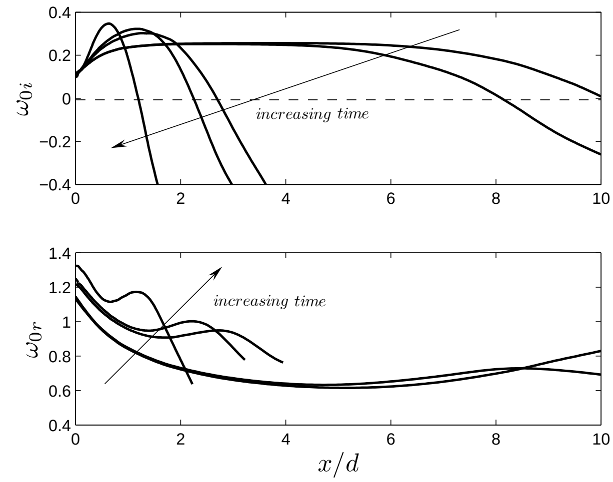

FIG. 4: Spatial evolution the local absolute growth rate (ω_{0i}) and local pulsation (ω_{0r}) as a function of the longitudinal coordinate x as time increases during the transitory (t=50, 66, 81, 88 and 110).

on the analytic continuation of $\omega_0(x)$ in the x-plane and given by $\omega_g = \mathcal{R}e(\omega_0(x_s))$ with $\frac{d\omega_0}{dx}(x_s) = 0$ and where x_s is a complex number and ω_g represents the global frequency selected by the wake deduced from local properties [10]. The method used to calculate this global frequency is detailed in [2]. Finally, we compare the numerical results with those of our theoretical prediction. These are plotted in figure 2 and we see clearly that a simple linear criterion closely follows the frequency obtained with our numerical simulation. Finally, figure (5) shows the evolution of the real part of the saddle point $\mathcal{R}e(x_s)$ giving the selected frequency as a function of time and indicates that the critical point responsible for the selection of the global frequency moves back close to cylinder. One can note that this evolution follows those observed for the recirculation zone observed in figure 3.

Thus, in this letter, we show the fundamental role played by the mean flow correction in the stability properties in fluid mechanics. The main important comment to be made is that the basic flow is not the right flow to be considered when predicting stability properties as it differs strongly from the real observed flow. The Bénard-von Kàrmàn instability is driven by the two unstable shear layers located on both sides of the body and it is clear that they are affected by the non-linear mean flow correction. The consequence is that the real flow has no link with the basic one. It seems that the mean flow, even if it is the result of a temporal averaging, is the closest flow to the real one which can be understood if we consider that modes are

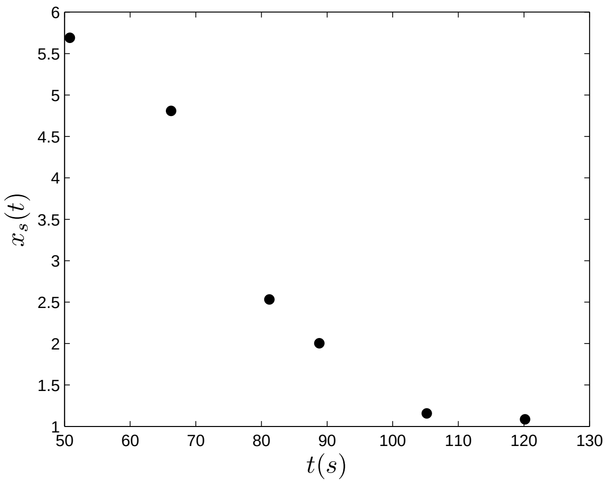

FIG. 5: Evolution of the saddle point location as a function of time during the transient regime of the wake.

selected linearly. The mean flow correction changes the equilibrium position (or stable fixed point) of the system by modifying the size and the intensity of the absolute region.

One has to note that theoretical arguments have to be put forward to establish clearly the validity of using linear stability on time averaged flow and this problem is now one of the preoccupation of the community [1–3]. However, the fact that stability properties are related to the mean state gives a new perspective in the study of such flows. For example, it is now possible to understand global behavior and mode selection in controlled or forced flows, for which it is established that the simple fact of step in a flow changes its mean state [2], but also determinate properties of large scales of turbulent flows, which should be dominated by the mean shear.

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[13] A linear eigenfunction is used to perturb the base flow. This eigenfunction is computed by subtracting the base flow (obtained by forcing the symmetry around the x-axis of the cavity at y=0) to the full flow pattern in the linear regime (starting from the basic flow, the numerical noise is sufficient to start the instability). This procedure is repeated at $t + nt_i$ for $t_i = n/100$ (being the period in the linear regime) and we finally obtain 100 eigenmodes, each with exponentially growing amplitude and different phase. We normalize the amplitude of the eigenmodes to 10^{-3} and thus have a wide base of initial perturbations.